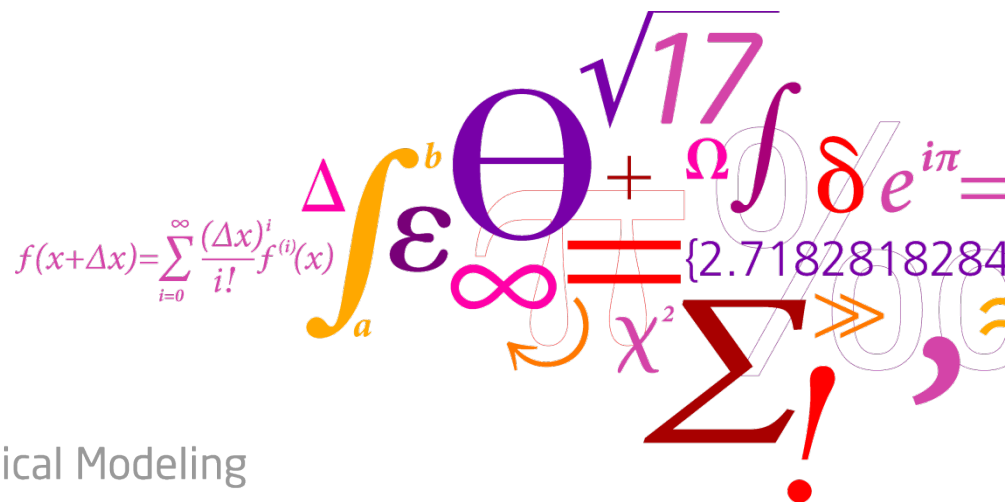


Tools for decision making under uncertainty in electricity markets

Lecture 6: “Risk management through forward contracts”

Juan Miguel Morales
Salvador Pineda



Outline

- CVaR
- Forward contracts
- Decision-making tool
 - Example
- Scenario reduction
 - Example

CVaR

- Determining the Value-at-Risk (VaR)

$$VaR_{\alpha}(\eta) = \min\{\xi : F_{\eta}(\xi) \geq 1 - \alpha\}$$

Scenario	Profit (\$)	Prob	Cum prob
w1	12	0.1	0.1
w2	16	0.1	0.2
w3	20	0.2	0.4
w4	30	0.2	0.6
w5	48	0.2	0.8
w6	51	0.2	1.0

Alfa	VaR (\$)
0	
0.2	
0.5	
0.8	

CVaR

- Determining the Value-at-Risk (VaR)

$$VaR_{\alpha}(\eta) = \min\{\xi : F_{\eta}(\xi) \geq 1 - \alpha\}$$

Scenario	Profit (\$)	Prob	Cum prob
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w2	16	0.1	0.2
w3	20	0.2	0.4
w4	30	0.2	0.6
w5	48	0.2	0.8
w6	51	0.2	1.0

Alfa	VaR (\$)
0	51
0.2	48
0.5	30
0.8	16

CVaR

- Determining the Conditional Value-at-Risk (CVaR), defined as the average value of the profits lower than the VaR.

Scenario	Profit (\$)	Prob	Cum prob
w1	12	0.1	0.1
w2	16	0.1	0.2
w3	20	0.2	0.4
w4	30	0.2	0.6
w5	48	0.2	0.8
w6	51	0.2	1.0

Alfa	VaR (\$)	CVaR (\$)
0	51	
0.2	48	
0.5	30	
0.8	16	

CVaR

- Determining the Conditional Value-at-Risk (CVaR), defined as the average value of the profits lower than the VaR.

Scenario	Profit (\$)	Prob	Cum prob
w1	12	0.1	0.1
w2	16	0.1	0.2
w3	20	0.2	0.4
w4	30	0.2	0.6
w5	48	0.2	0.8
w6	51	0.2	1.0

Alfa	VaR (\$)	CVaR (\$)
0	51	32.6
0.2	48	28.0
0.5	30	19.6
0.8	16	14

CVaR

- Optimization problem

$$CVaR_{\alpha=0.5} = \text{Maximize} \quad \xi - \frac{1}{1-\alpha} \sum_{w=1}^W \pi_w \eta_w$$

$$S.T. \quad \eta_w \geq -\Pi_w + \xi \quad \forall w$$

$$\eta_w \geq 0 \quad \forall w$$

ξ^* (\$)	o.f. (\$)
12	

CVaR

- Optimization problem

$$CVaR_{\alpha=0.5} = \text{Maximize} \quad \xi - \frac{1}{1-\alpha} \sum_{w=1}^W \pi_w \eta_w$$

$$S.T. \quad \eta_w \geq -\Pi_w + \xi \quad \forall w$$

$$\eta_w \geq 0 \quad \forall w$$

Scenario	η_w
w1	0
w2	0
w3	0
w4	0
w5	0
w6	0

ξ^* (\$)	o.f. (\$)
12	

CVaR

- Optimization problem

$$CVaR_{\alpha=0.5} = \text{Maximize} \quad \xi - \frac{1}{1-\alpha} \sum_{w=1}^W \pi_w \eta_w$$

$$S.T. \quad \eta_w \geq -\Pi_w + \xi \quad \forall w$$

$$\eta_w \geq 0 \quad \forall w$$

Scenario	η_w
w1	0
w2	0
w3	0
w4	0
w5	0
w6	0

ξ^* (\$)	o.f. (\$)
12	12.0

CVaR

- Optimization problem

$$CVaR_{\alpha=0.5} = \text{Maximize} \quad \xi - \frac{1}{1-\alpha} \sum_{w=1}^W \pi_w \eta_w$$

$$S.T. \quad \eta_w \geq -\Pi_w + \xi \quad \forall w$$

$$\eta_w \geq 0 \quad \forall w$$

ξ^* (\$)	o.f. (\$)
12	12.0
16	

CVaR

- Optimization problem

$$CVaR_{\alpha=0.5} = \text{Maximize} \quad \xi - \frac{1}{1-\alpha} \sum_{w=1}^W \pi_w \eta_w$$

$$S.T. \quad \eta_w \geq -\Pi_w + \xi \quad \forall w$$

$$\eta_w \geq 0 \quad \forall w$$

Scenario	η_w
w1	4
w2	0
w3	0
w4	0
w5	0
w6	0

ξ^* (\$)	o.f. (\$)
12	12.0
16	

CVaR

- Optimization problem

$$CVaR_{\alpha=0.5} = \text{Maximize} \quad \xi - \frac{1}{1-\alpha} \sum_{w=1}^W \pi_w \eta_w$$

$$S.T. \quad \eta_w \geq -\Pi_w + \xi \quad \forall w$$

$$\eta_w \geq 0 \quad \forall w$$

Scenario	η_w
w1	4
w2	0
w3	0
w4	0
w5	0
w6	0

ξ^* (\$)	o.f. (\$)
12	12.0
16	15.2

CVaR

- Optimization problem

$$CVaR_{\alpha=0.5} = \text{Maximize} \quad \xi - \frac{1}{1-\alpha} \sum_{w=1}^W \pi_w \eta_w$$

$$S.T. \quad \eta_w \geq -\Pi_w + \xi \quad \forall w$$

$$\eta_w \geq 0 \quad \forall w$$

Alfa	VaR (\$)	CVaR (\$)
0	51	32.6
0.2	48	28.0
0.5	30	19.6
0.8	16	14

ξ^* (\$)	o.f. (\$)
12	12.0
16	15.2
20	17.6
30	19.6
48	16.0
51	14.2

Forward contracts

- Definition
 - Quantity of energy
 - Delivery period
 - Fixed price
 - Profit

$$\Pi^F = \sum_{c=1}^{N_C} \lambda_{(c)} P_{(c)} L_{(c)}$$

Forward contracts

	SPANISH POWER	
	Base	Peak
YR-13	51.20	55.47
YR-14	50.60	54.87
Q3-12	55.50	59.74
Q4-12	53.55	58.01
Q1-13	53.44	57.71
Q2-13	48.47	52.75
Q3-13	52.55	56.81
Q4-13	50.36	54.63
M May-12	50.65 ↓	55.27
M Jun-12	55.33	60.34
M Jul-12	56.70	60.94
M Aug-12	54.78	58.98
M Sep-12	55.00	59.29
Wk17-12	47.23	52.23
Wk18-12	48.00	52.88
Wk19-12	50.25	55.27

Production cost (€/MWh)	20
Maximum capacity (MW)	150
Minimum output (MW)	50

Price (€/MWh)	d1	d2	d3	d4	d5	d6	d7
	52	52	52	52	52	52	52

Pool?	Contracts?

Forward contracts

	SPANISH POWER	
	Base	Peak
YR-13	51.20	55.47
YR-14	50.60	54.87
Q3-12	55.50	59.74
Q4-12	53.55	58.01
Q1-13	53.44	57.71
Q2-13	48.47	52.75
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Wk18-12	48.00	52.88
Wk19-12	50.25	55.27

Production cost (€/MWh)	20
Maximum capacity (MW)	150
Minimum output (MW)	50

Price (€/MWh)	d1	d2	d3	d4	d5	d6	d7
	52	52	52	52	52	52	52

Pool?	Contracts?
150	0

Forward contracts

	SPANISH POWER	
	Base	Peak
YR-13	51.20	55.47
YR-14	50.60	54.87
Q3-12	55.50	59.74
Q4-12	53.55	58.01
Q1-13	53.44	57.71
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Wk17-12	47.23	52.23
Wk18-12	48.00	52.88
Wk19-12	50.25	55.27

Production cost (€/MWh)	20
Maximum capacity (MW)	150
Minimum output (MW)	50

Price (€/MWh)	d1	d2	d3	d4	d5	d6	d7
	47	47	47	47	47	47	47

Pool?	Contracts?

Forward contracts

	SPANISH POWER	
	Base	Peak
YR-13	51.20	55.47
YR-14	50.60	54.87
Q3-12	55.50	59.74
Q4-12	53.55	58.01
Q1-13	53.44	57.71
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Wk17-12	47.23	52.23
Wk18-12	48.00	52.88
Wk19-12	50.25	55.27

Production cost (€/MWh)	20
Maximum capacity (MW)	150
Minimum output (MW)	50

Price (€/MWh)	d1	d2	d3	d4	d5	d6	d7
	47	47	47	47	47	47	47

Pool?	Contracts?
0	150

Optimization problem?

Forward contracts

- Optimization problem

$$\text{Maximize} \quad \lambda_c P_c T_c + \sum_{t=1}^T \left(\lambda_t^P P_t^P - C_t P_t^G \right)$$

$$S.T. \quad P^{\min} \leq P_t^G \leq P^{\max} \quad \forall t$$

$$P_t^G = P_t^P + P_c \quad \forall t$$

$$P_t^P \geq 0 \quad \forall t$$

$$P_c \geq 0$$

Forward contracts

SPANISH POWER

	Base	Peak
YR-13	51.20	55.47
YR-14	50.60	54.87
Q3-12	55.50	59.74
Q4-12	53.55	58.01
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Wk17-12	47.23	52.23
Wk18-12	48.00	52.88
Wk19-12	50.25	55.27

Production cost (€/MWh)	20
Maximum capacity (MW)	150
Minimum output (MW)	50

	d1	d2	d3	d4	d5	d6	d7
Price (€/MWh)	52	52	52	52	52	52	52
	47	47	47	47	47	47	47

Average price 49.5 €/MWh

Pool?	Contracts?

Depends on
risk-aversion!

Forward contracts

- Optimization problem

$$\text{Maximize} \quad \xi - \frac{1}{1-\alpha} \sum_{w=1}^W \pi_w \eta_w$$

$$\text{S.T.} \quad \Pi_w = \lambda_c P_c T_c + \sum_{t=1}^T \left(\lambda_{tw}^P P_{tw}^P - C_t P_{tw}^G \right) \quad \forall w$$

$$P_{tw}^G = P_{tw}^P + P_c \quad \forall t, \forall w$$

$$P^{\min} \leq P_{tw}^G \leq P^{\max} \quad \forall t, \forall w$$

$$\eta_w \geq -\Pi_w + \xi \quad \forall w$$

$$\eta_w \geq 0 \quad \forall w$$

$$P_t^P \geq 0 \quad \forall t$$

$$P_c \geq 0$$

Forward contracts

SPANISH POWER

	Base	Peak
YR-13	51.20	55.47
YR-14	50.60	54.87
Q3-12	55.50	59.74
Q4-12	53.55	58.01
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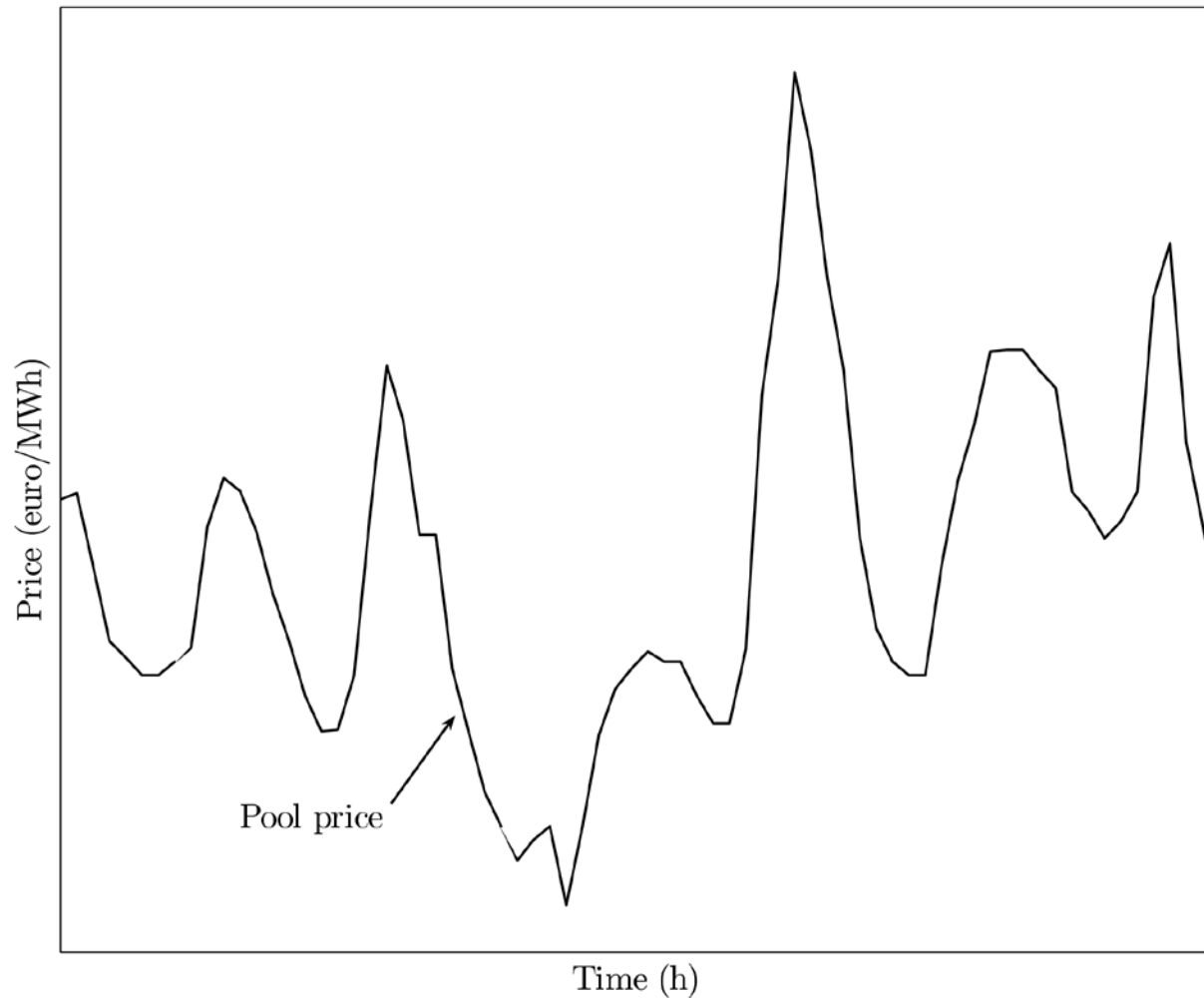
Production cost (€/MWh)	20
Maximum capacity (MW)	150
Minimum output (MW)	50

	d1	d2	d3	d4	d5	d6	d7
Price (€/MWh)	52	52	52	52	52	52	52
	47	47	47	47	47	47	47

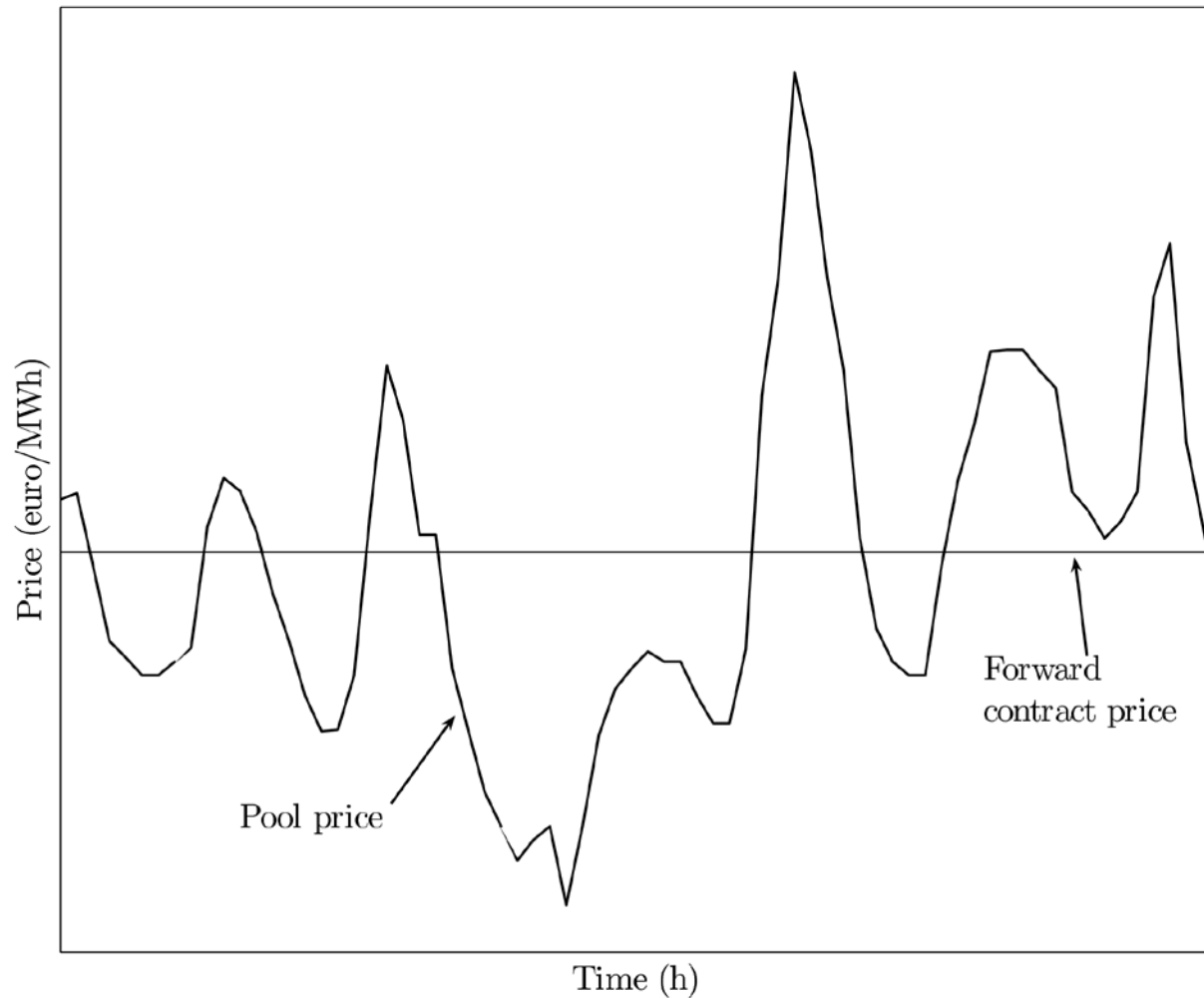
Average price 49.5 €/MWh

Alfa	Pool?	Contracts?
0	150	0
0.95	0	150

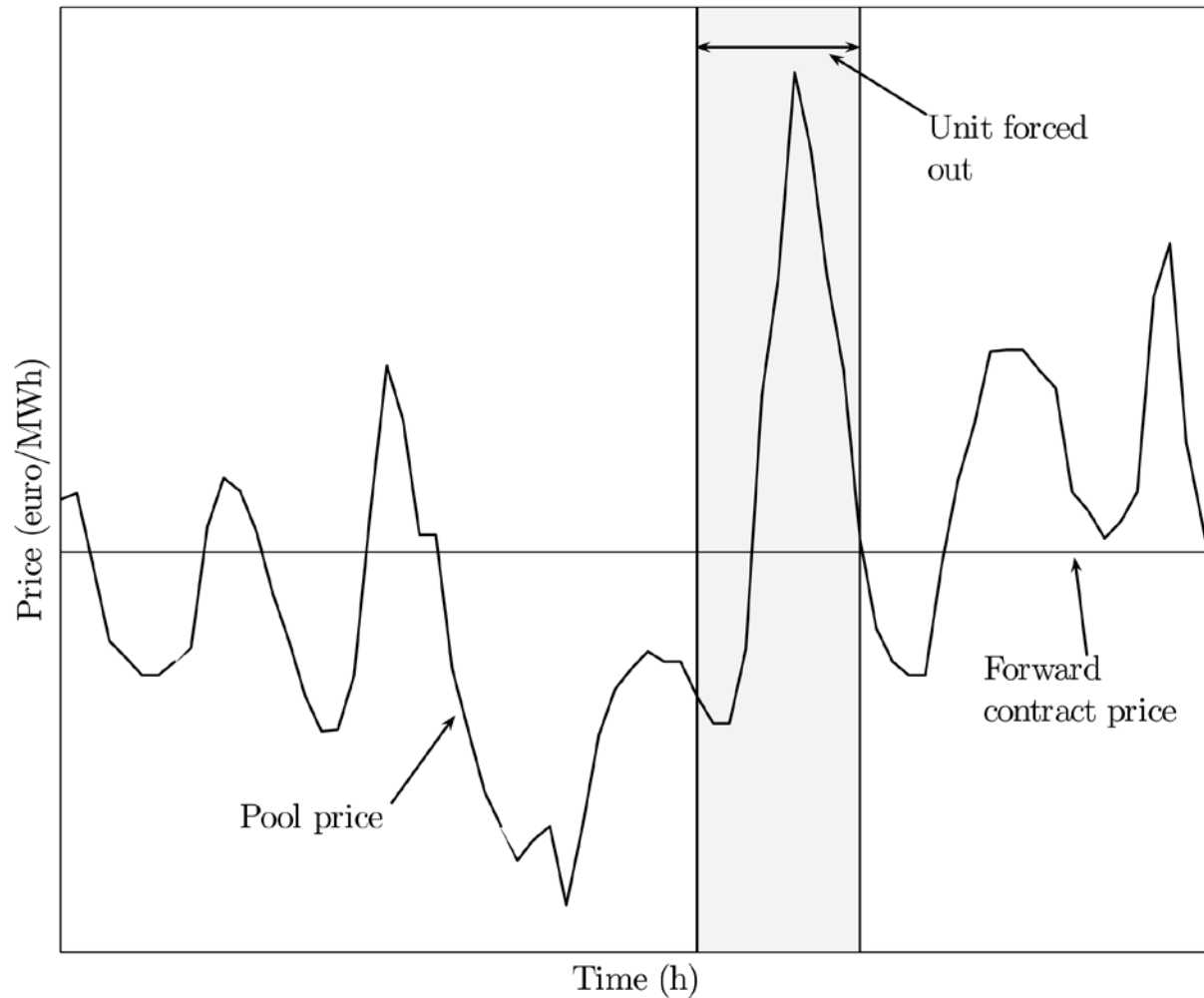
Forward contracts



Forward contracts



Forward contracts



Forward contracts

Wk17-12	47.23	52.23
Wk18-12	48.00	52.88
Wk19-12	50.25	55.27

Production cost (€/MWh)	20
Maximum capacity (MW)	150
Minimum output (MW)	50

Availability (FOR=0%)						
d1	d2	d3	d4	d5	d6	d7
1	1	1	1	1	1	1

Price (€/MWh)						
d1	d2	d3	d4	d5	d6	d7
52	52	52	52	52	52	52
47	47	47	47	47	47	47

	FOR = 0%
Alfa	Contracts
0	0
0.95	150

Forward contracts

Wk17-12	47.23	52.23
Wk18-12	48.00	52.88
Wk19-12	50.25	55.27

Production cost (€/MWh)	20
Maximum capacity (MW)	150
Minimum output (MW)	50

Availability (FOR=10%)						
d1	d2	d3	d4	d5	d6	d7
1	1	1	1	1	1	1
0	0	0	0	0	0	0

Price (€/MWh)						
d1	d2	d3	d4	d5	d6	d7
52	52	52	52	52	52	52
47	47	47	47	47	47	47

	FOR = 0%	FOR = 10%
Alfa	Contracts	Contracts
0	0	0
0.95	150	0

Forward contracts

- Optimization problem

$$\text{Maximize} \quad \xi - \frac{1}{1-\alpha} \sum_{w=1}^W \pi_w \eta_w$$

$$\text{S.T.} \quad \Pi_w = \lambda_c P_c T_c + \sum_{t=1}^T \left(\lambda_{tw}^P P_{tw}^P - C_t P_{tw}^G \right) \quad \forall w$$

$$P_{tw}^G = P_{tw}^P + P_c \quad \forall t, \forall w$$

$$P^{\min} \leq P_{tw}^G \leq P^{\max} \quad \forall t, \forall w$$

$$\eta_w \geq -\Pi_w + \xi \quad \forall w$$

$$\eta_w \geq 0 \quad \forall w$$

$$P_t^P \geq 0 \quad \forall t$$

$$P_c \geq 0$$

Forward contracts

- Optimization problem

$$\text{Maximize} \quad \xi - \frac{1}{1-\alpha} \sum_{w=1}^W \pi_w \eta_w$$

$$\text{S.T.} \quad \Pi_w = \lambda_c P_c T_c + \sum_{t=1}^T \left(\lambda_{tw}^P P_{tw}^P - C_t P_{tw}^G \right) \quad \forall w$$

$$P_{tw}^G = P_{tw}^P + P_c \quad \forall t, \forall w$$

$$k_{tw} P^{\min} \leq P_{tw}^G \leq k_{tw} P^{\max} \quad \forall t, \forall w$$

$$\eta_w \geq -\Pi_w + \xi \quad \forall w$$

$$\eta_w \geq 0 \quad \forall w$$

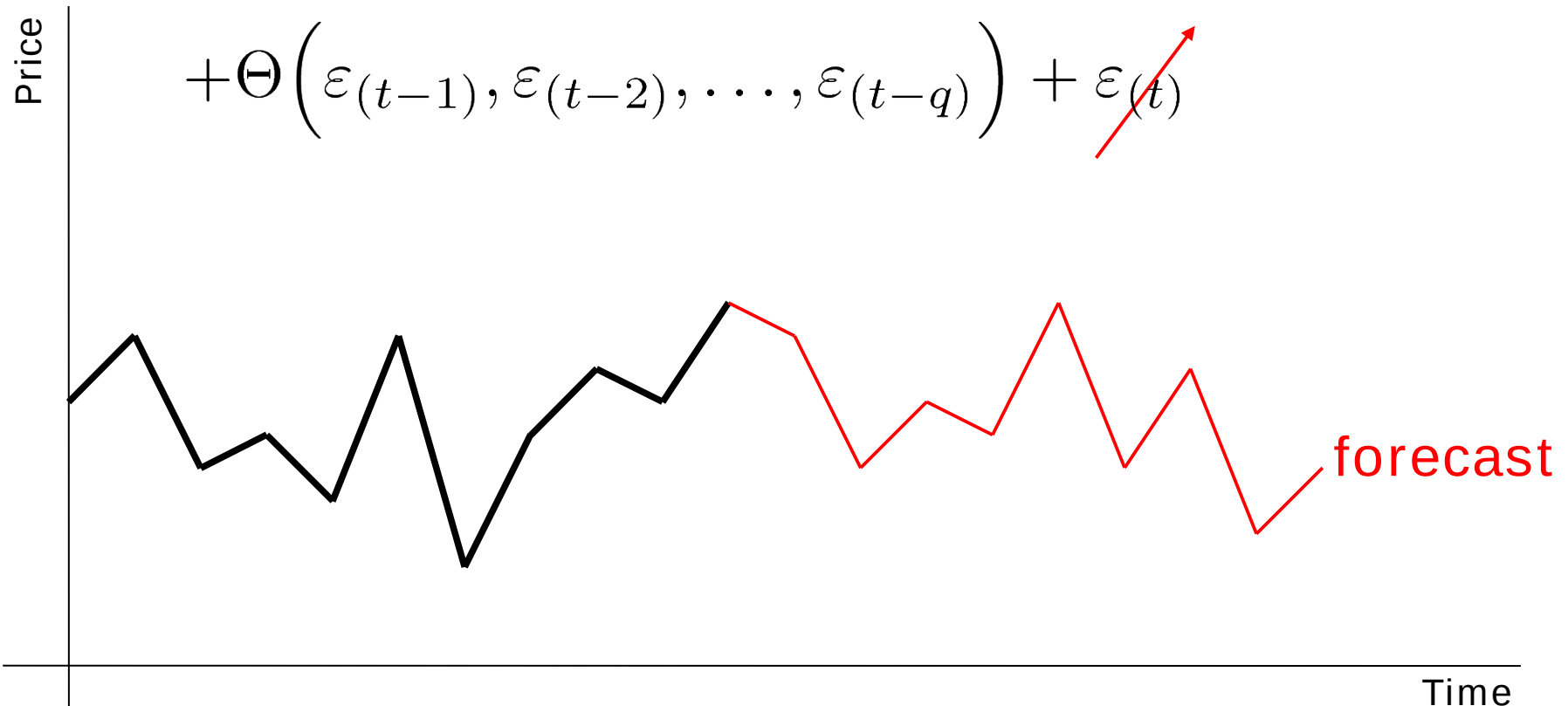
$$P_t^P \geq -(1 - k_{tw}) P^{\max} \quad \forall t$$

$$P_c \geq 0$$

Scenario generation

- ARIMA

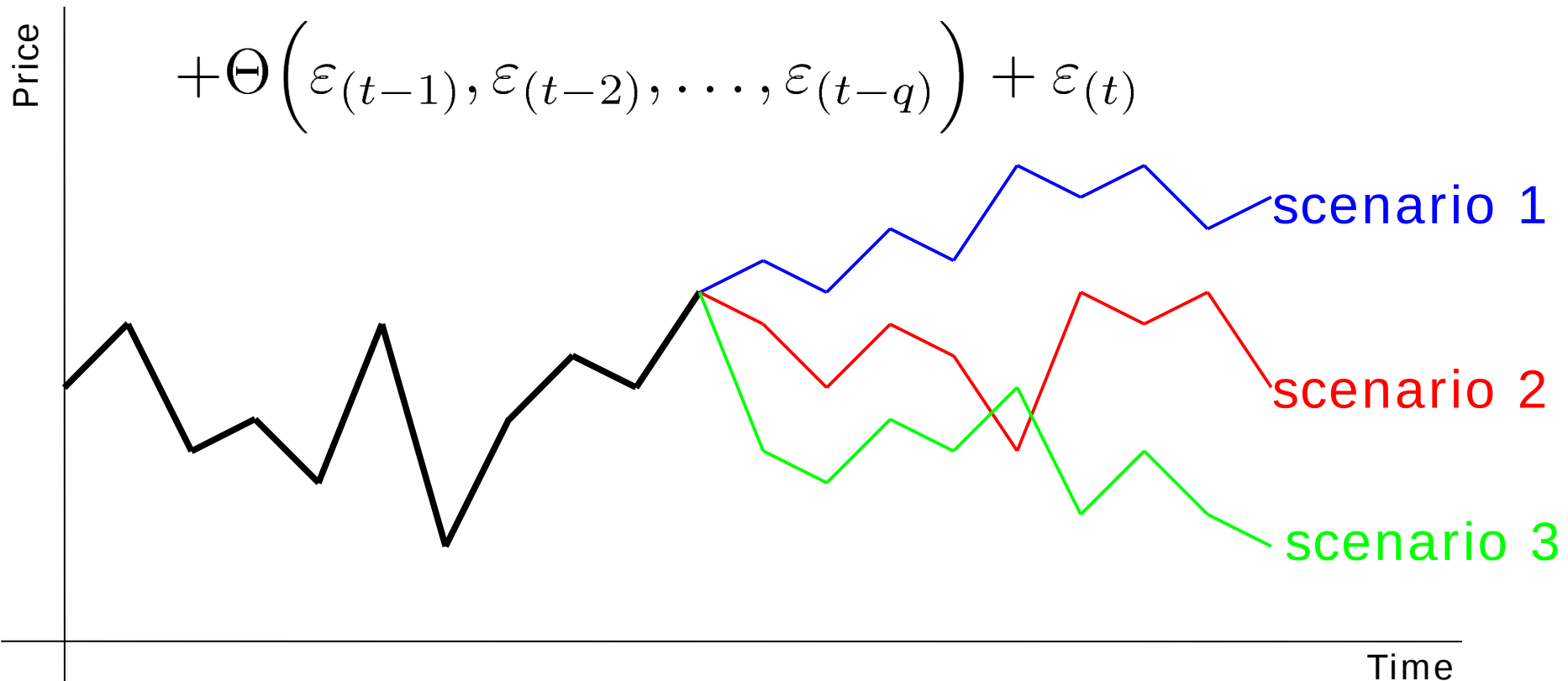
$$\widehat{\lambda^P}_{(t)} = \Phi\left(\lambda^P_{(t-1)}, \lambda^P_{(t-2)}, \dots, \lambda^P_{(t-p)}\right) + \\ + \Theta\left(\varepsilon_{(t-1)}, \varepsilon_{(t-2)}, \dots, \varepsilon_{(t-q)}\right) + \varepsilon_{(t)}$$



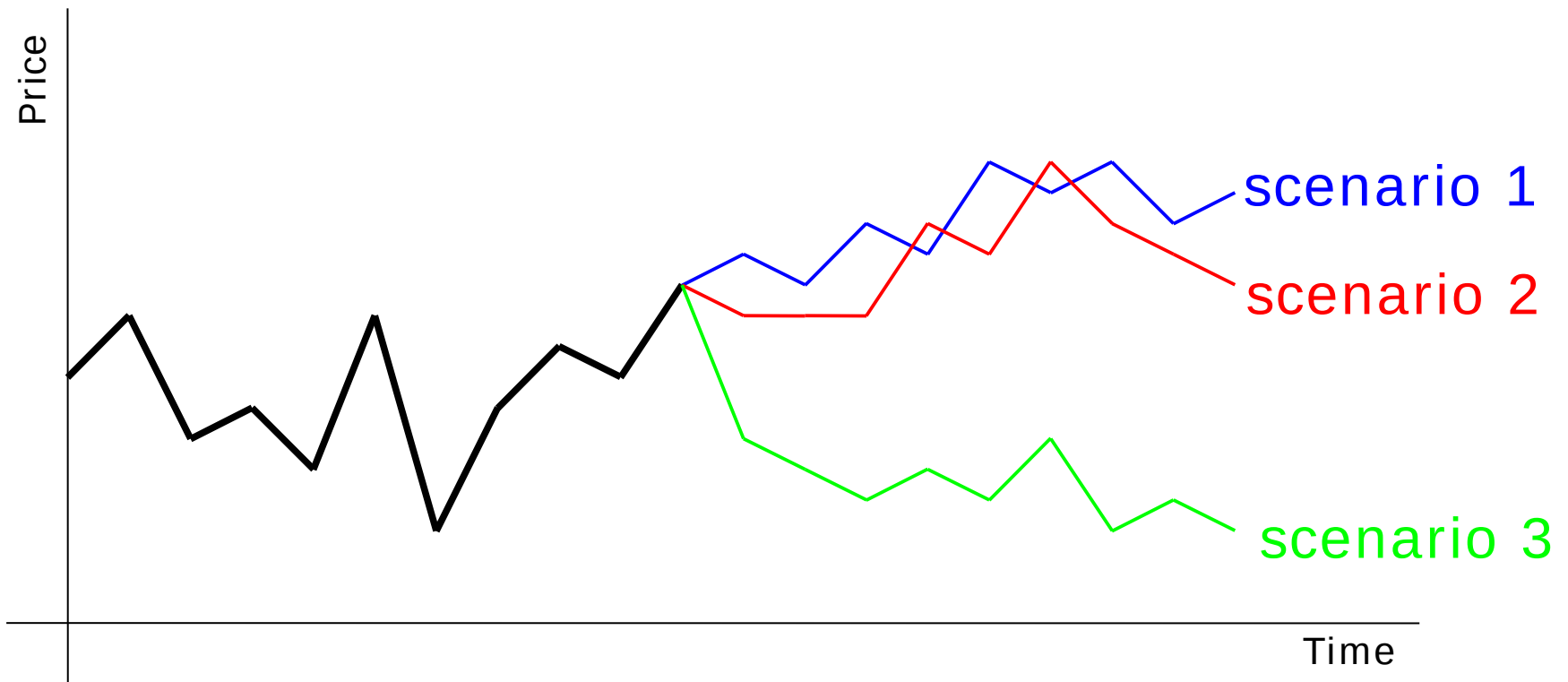
Scenario generation

- ARIMA

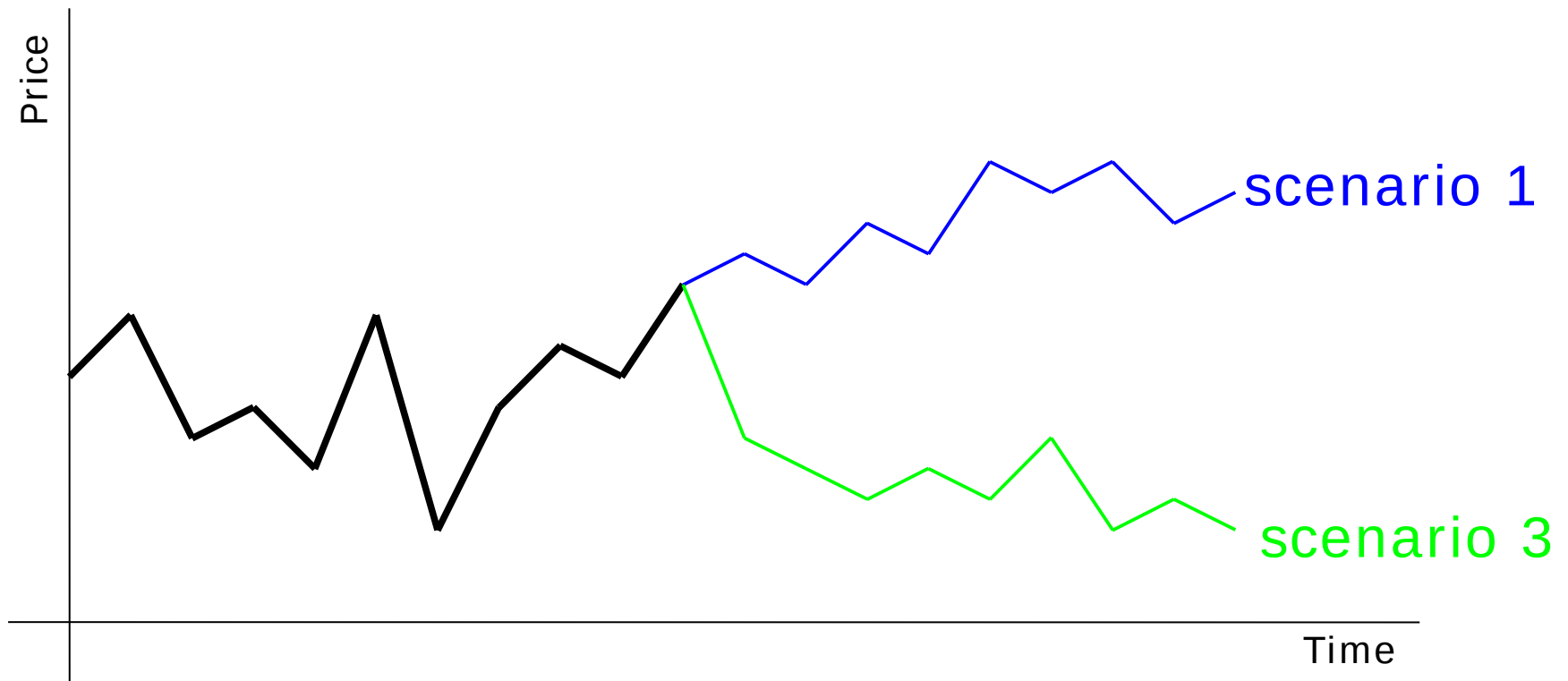
$$\widehat{\lambda^P}_{(t)} = \Phi\left(\lambda^P_{(t-1)}, \lambda^P_{(t-2)}, \dots, \lambda^P_{(t-p)}\right) + \\ + \Theta\left(\varepsilon_{(t-1)}, \varepsilon_{(t-2)}, \dots, \varepsilon_{(t-q)}\right) + \varepsilon_{(t)}$$



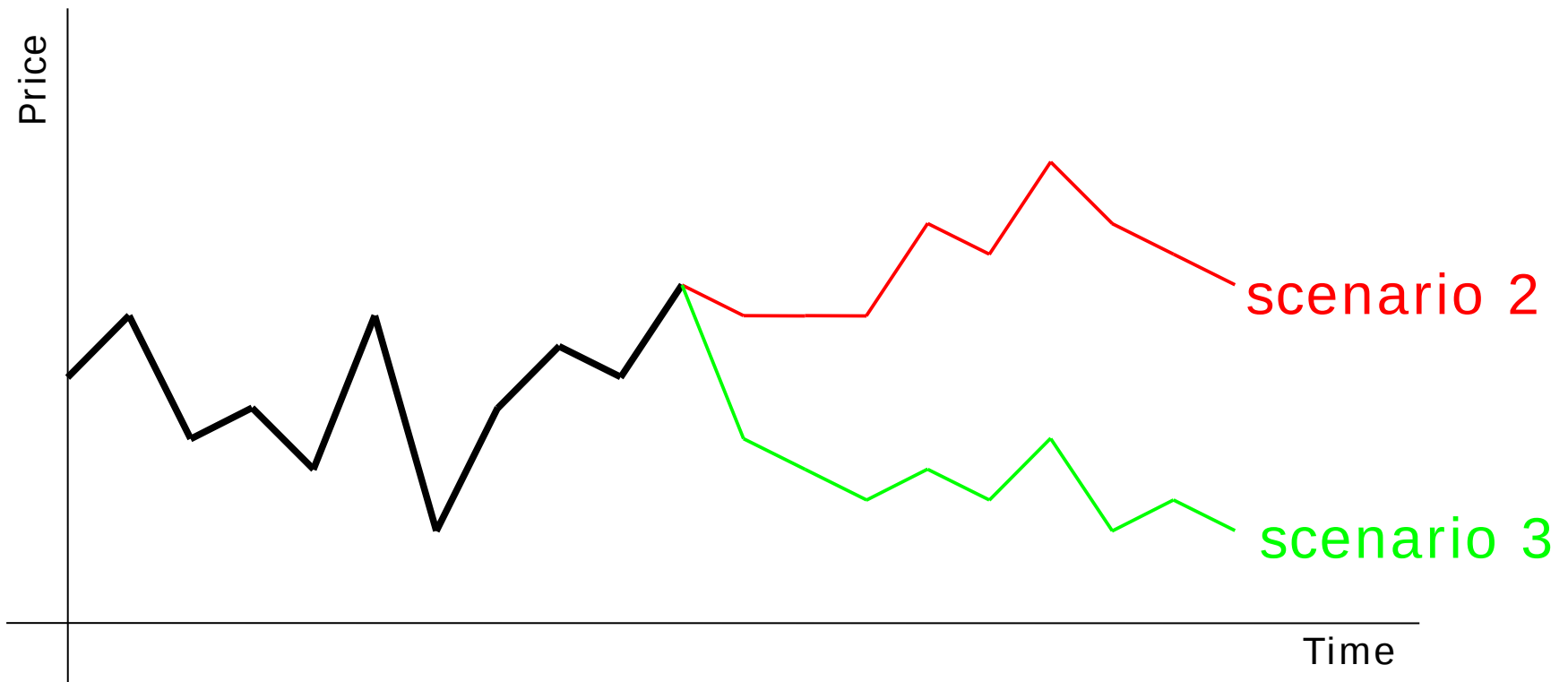
Scenario reduction



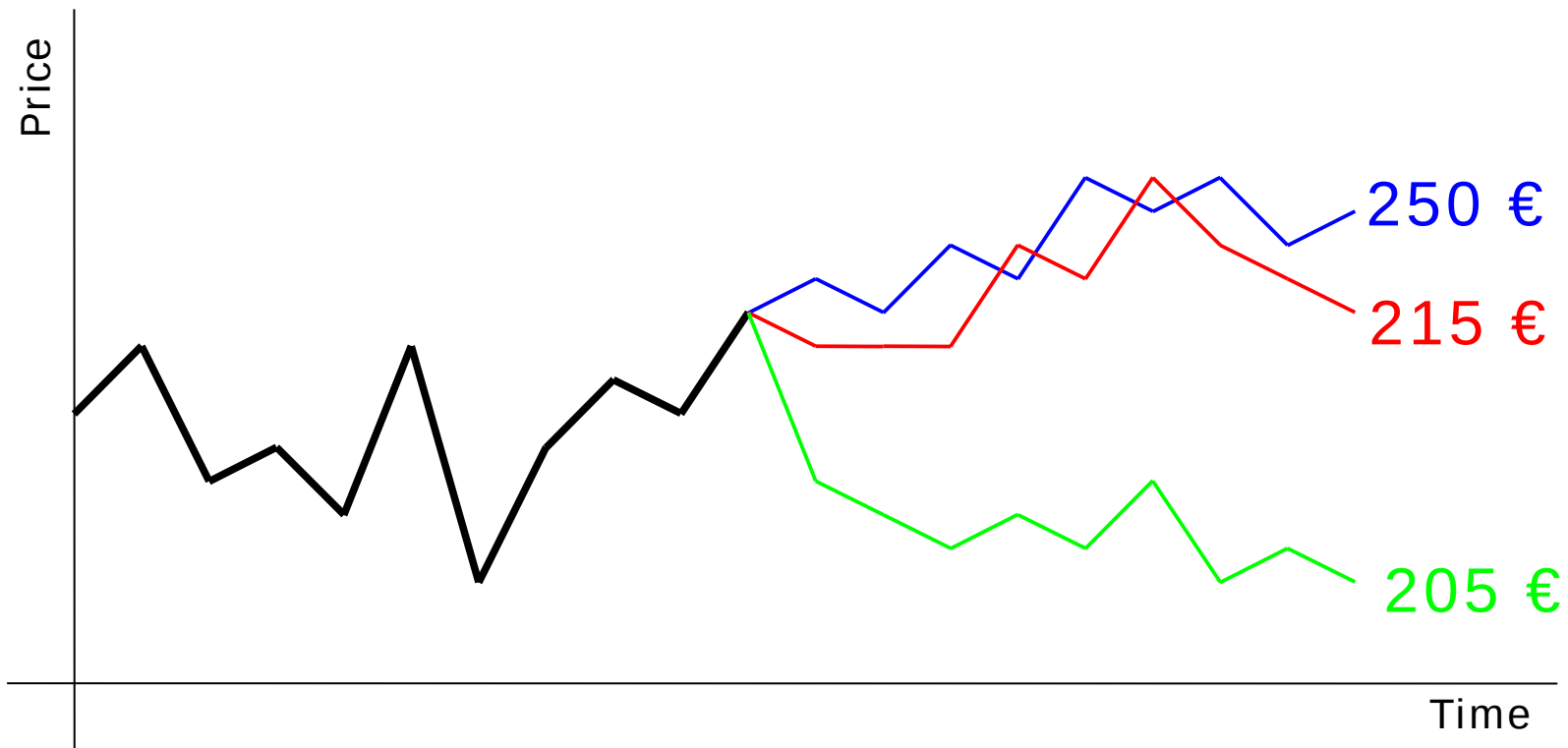
Scenario reduction



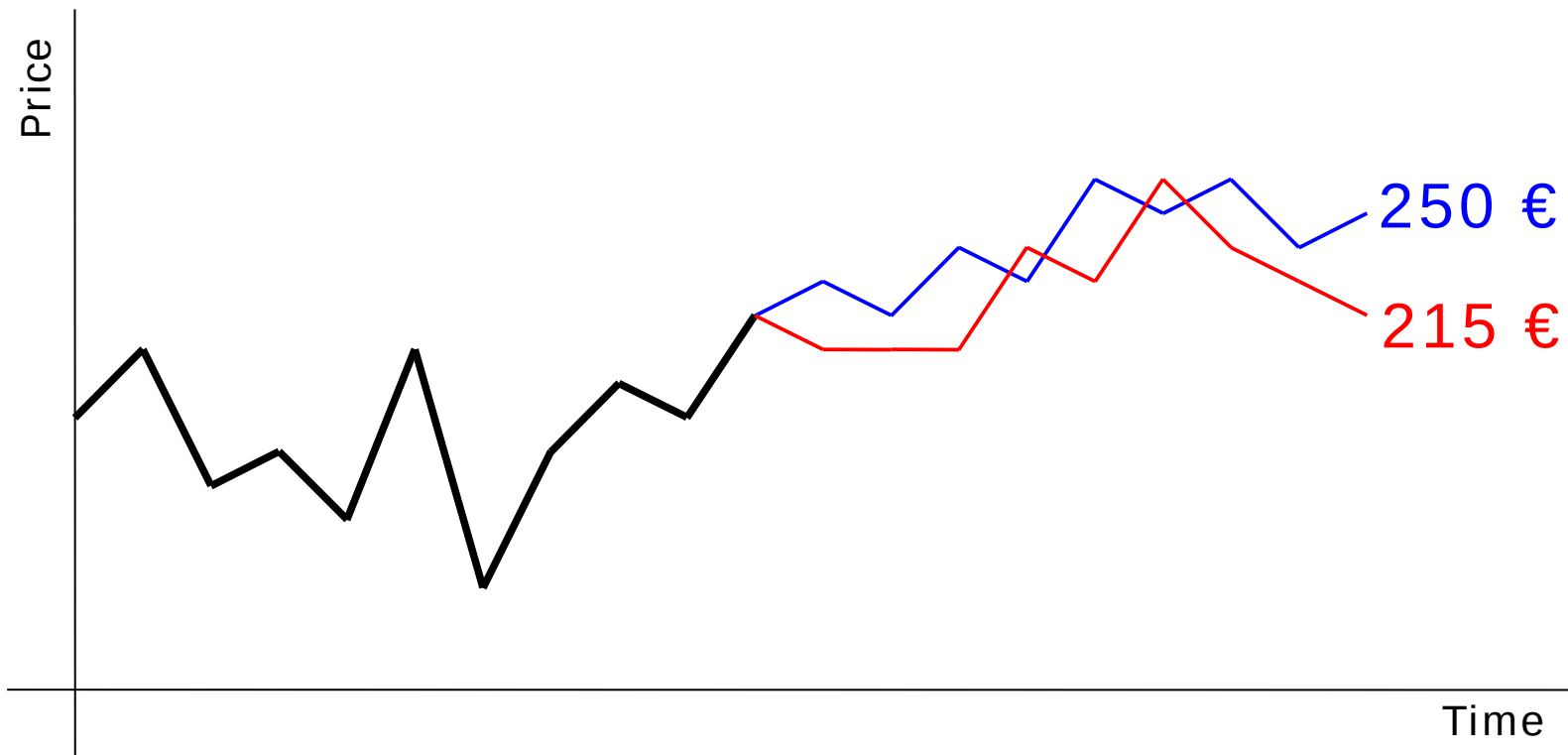
Scenario reduction



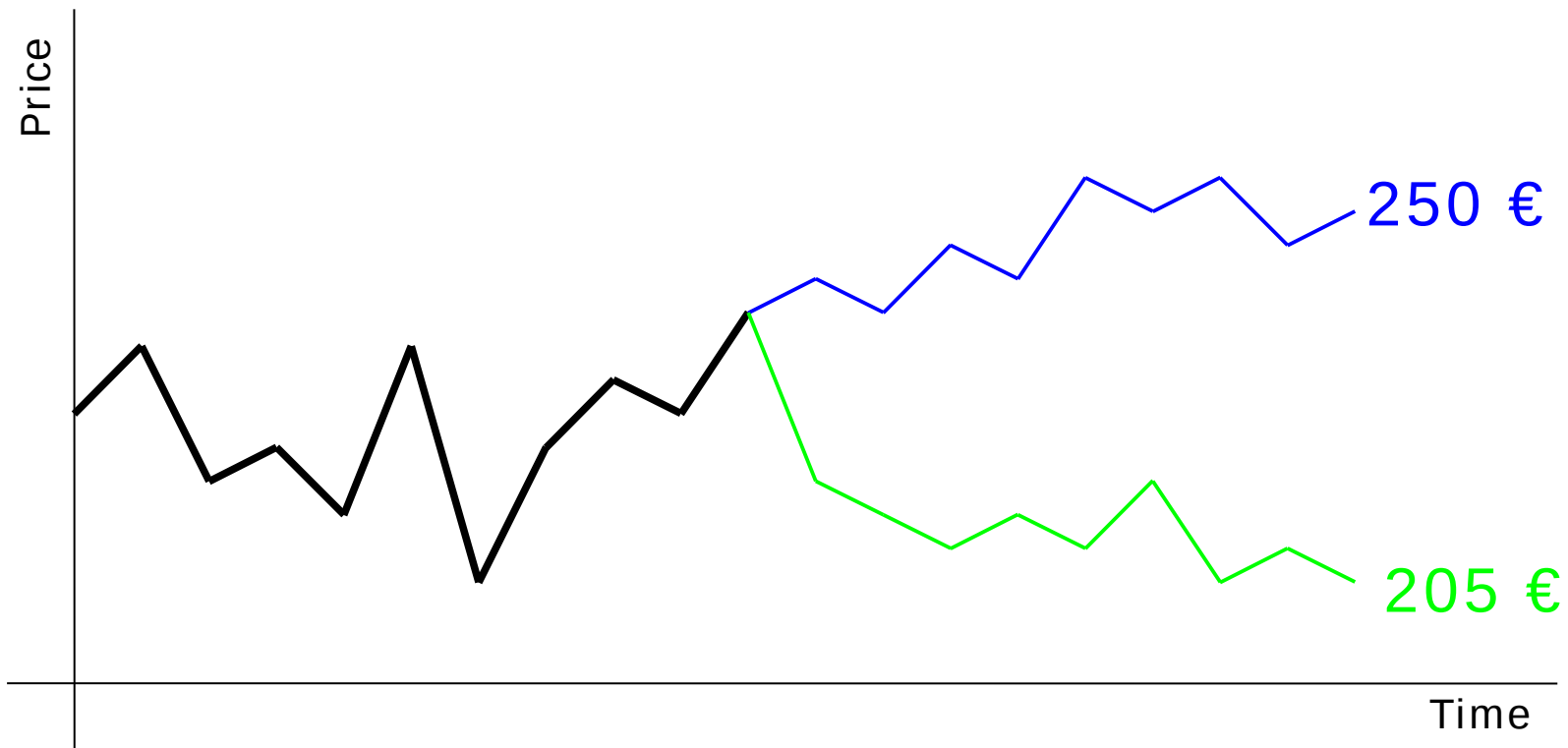
Scenario reduction



Scenario reduction



Scenario reduction

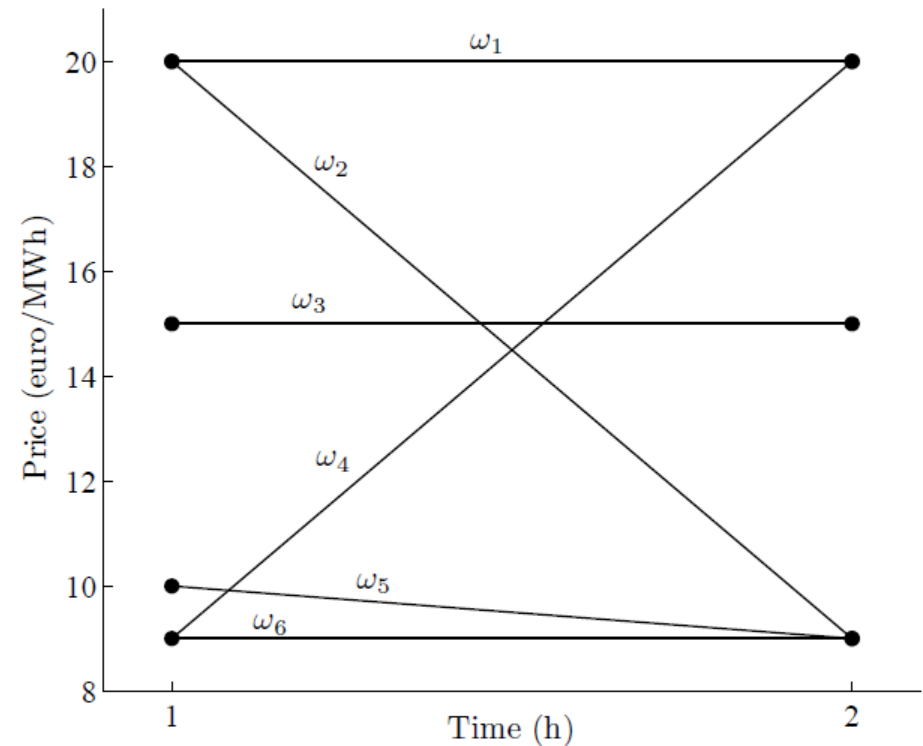


Scenario reduction

Production cost (€/MWh)	8
Maximum capacity (MW)	150
Minimum output (MW)	0

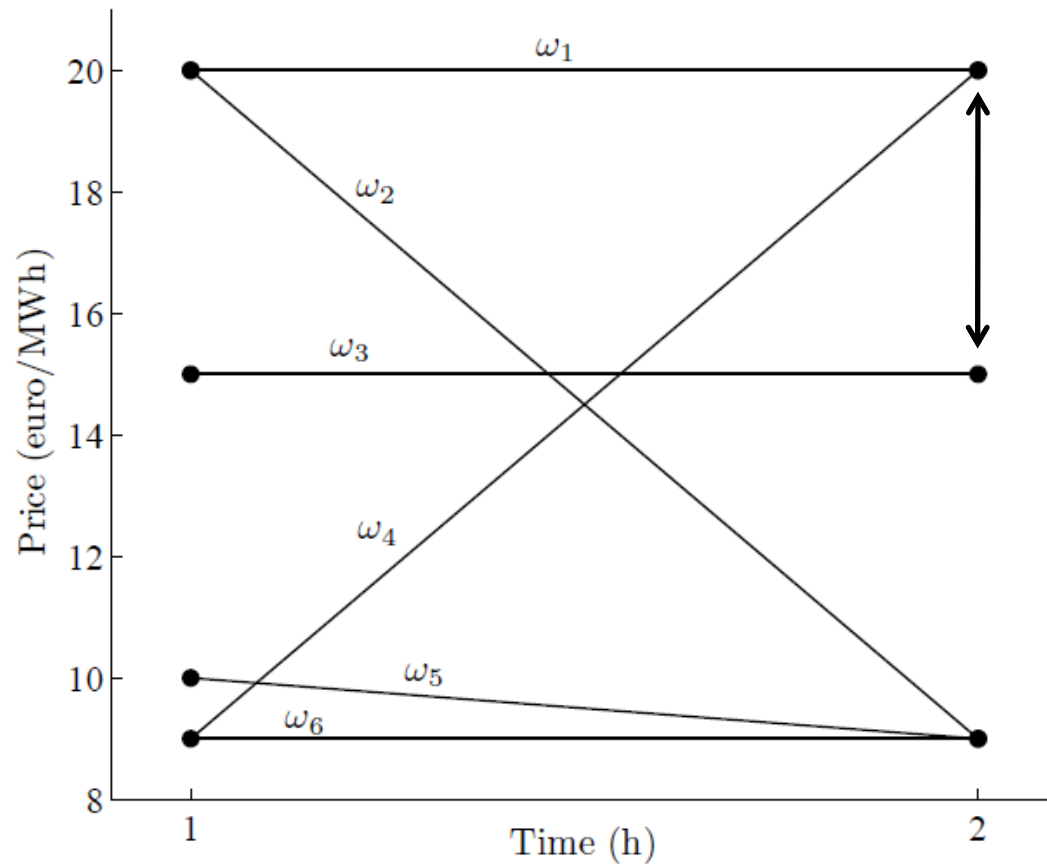
Scenario	t1	t2
w1	20	20
w2	20	9
w3	15	15
w4	9	20
w5	10	9
w6	9	9

Contract	Price	Pmax
c1	13.5	50
c2	9.5	50



Scenario reduction

- Technique A



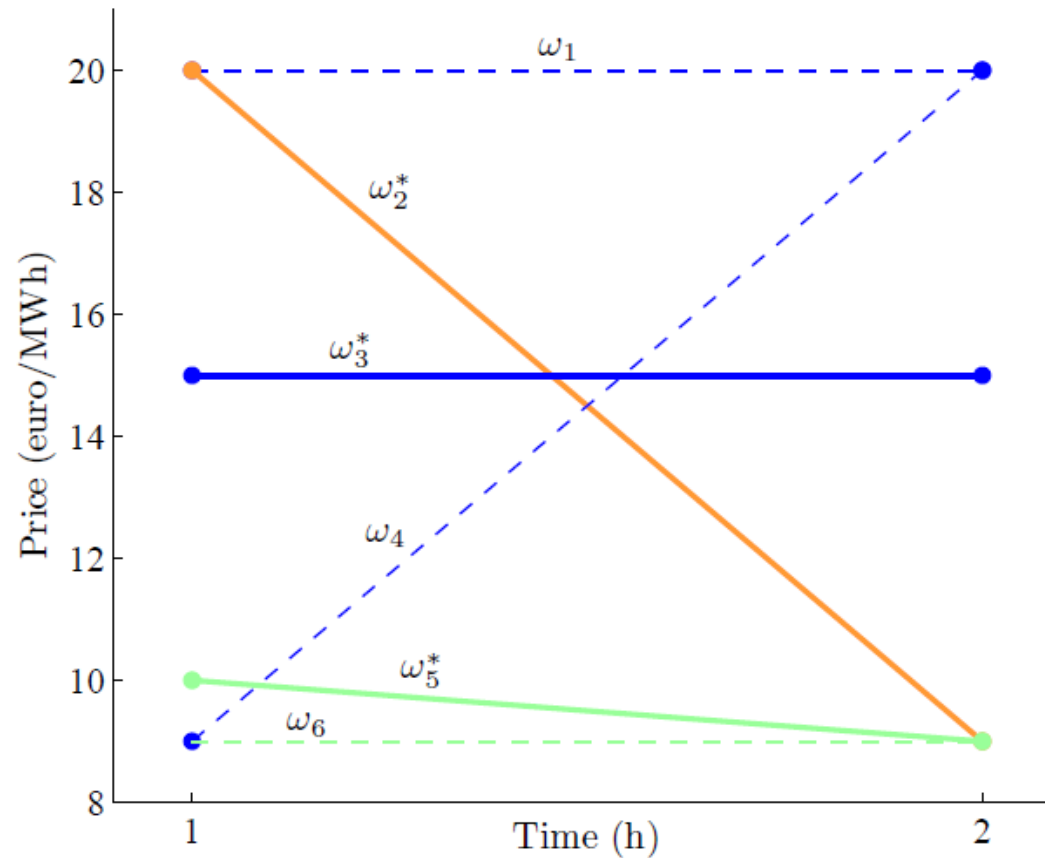
Scenario reduction

- Technique A
 - Distance between two scenarios

$$c(\omega, \omega') = \|\xi_{\omega} - \xi_{\omega'}\|$$

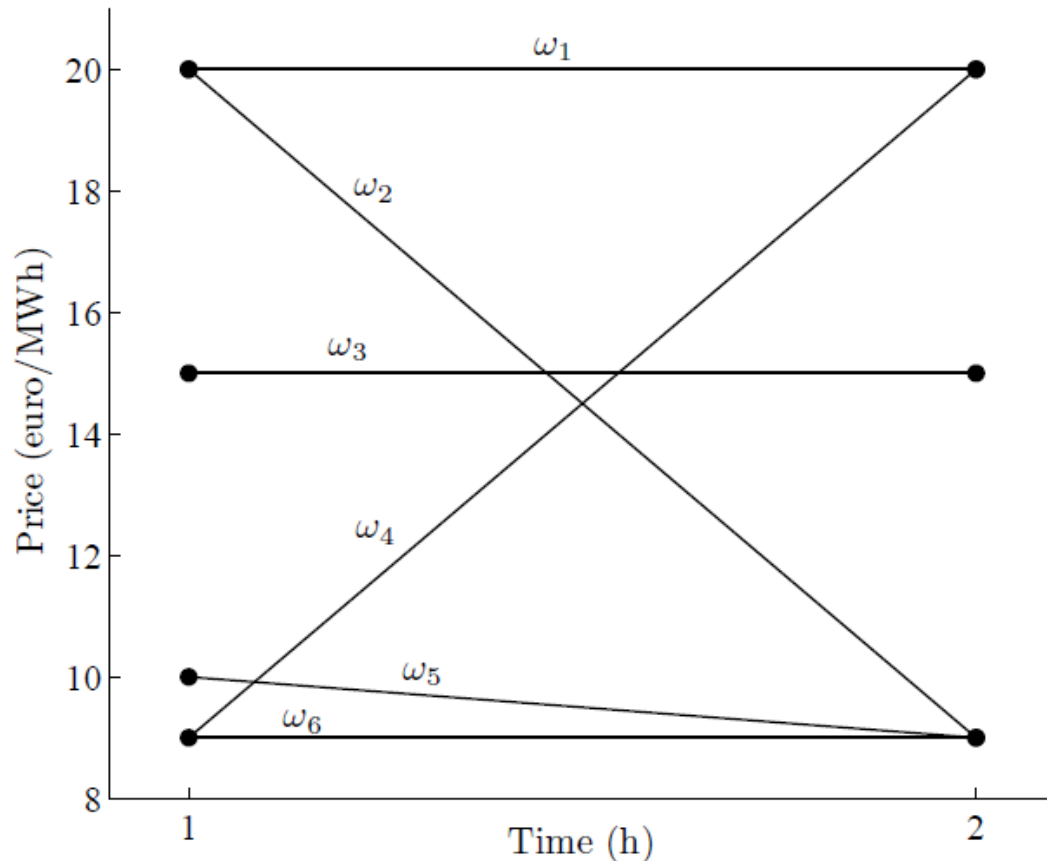
Scenario reduction

- Technique A



Scenario reduction

■ Technique B

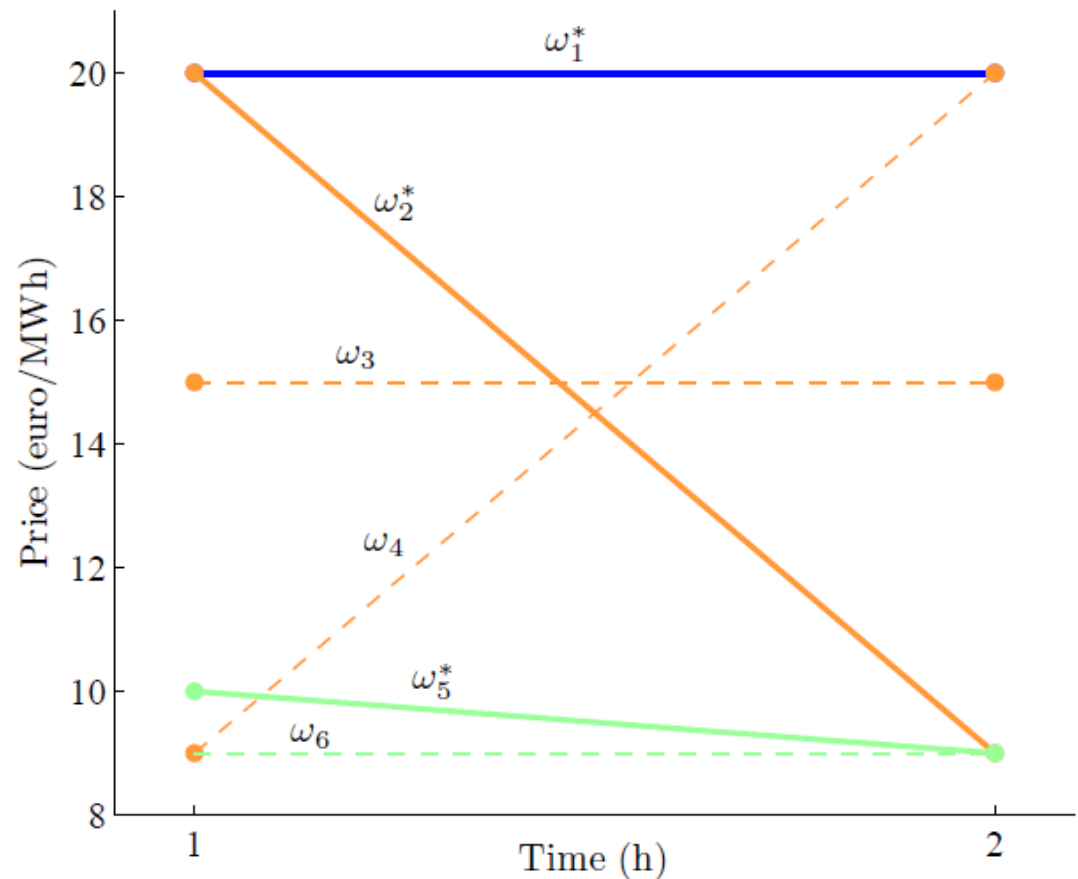


Scenario	Profit
w1	3600
w2	1950
w3	2100
w4	1950
w5	450
w6	300

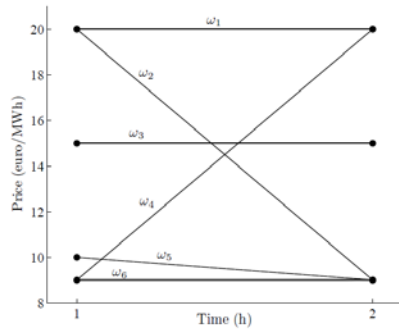
Scenario reduction

■ Technique B

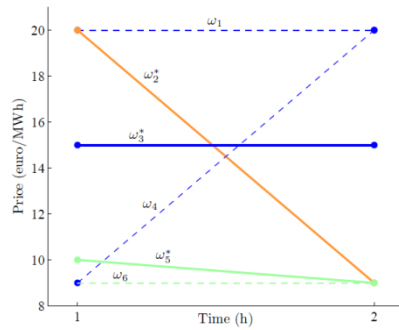
Scenario	Profit
w1	3600
w2	1950
w3	2100
w4	1950
w5	450
w6	300



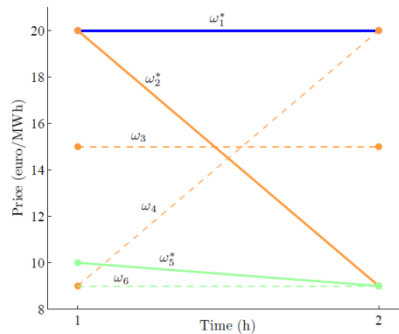
Scenario reduction



→ Profit (6 scenarios) = 1725 €

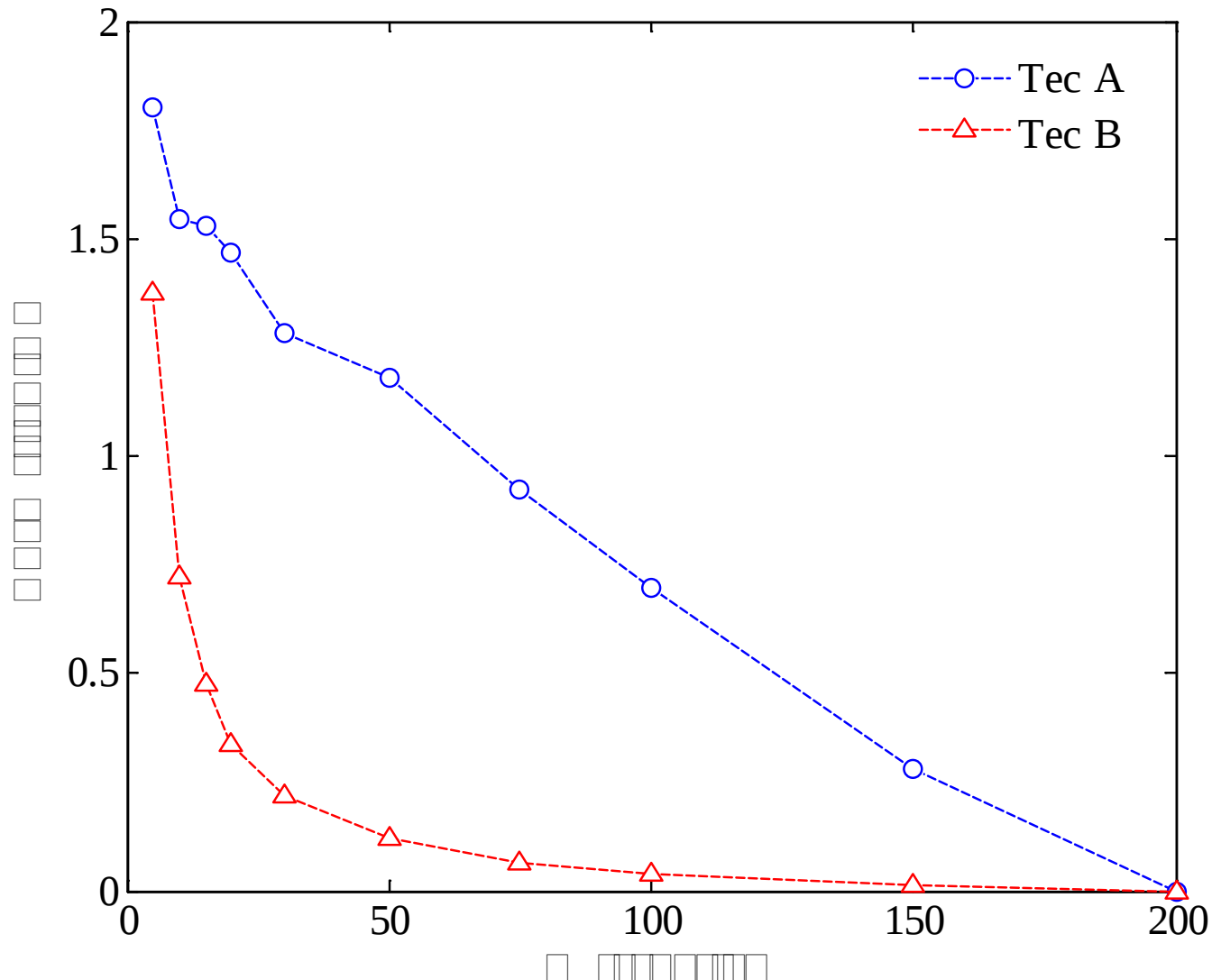


→ Profit (3 scenarios Technique A) = 1700 €



→ Profit (3 scenarios Technique B) = 1725 €

Scenario reduction





Thanks for your attention!